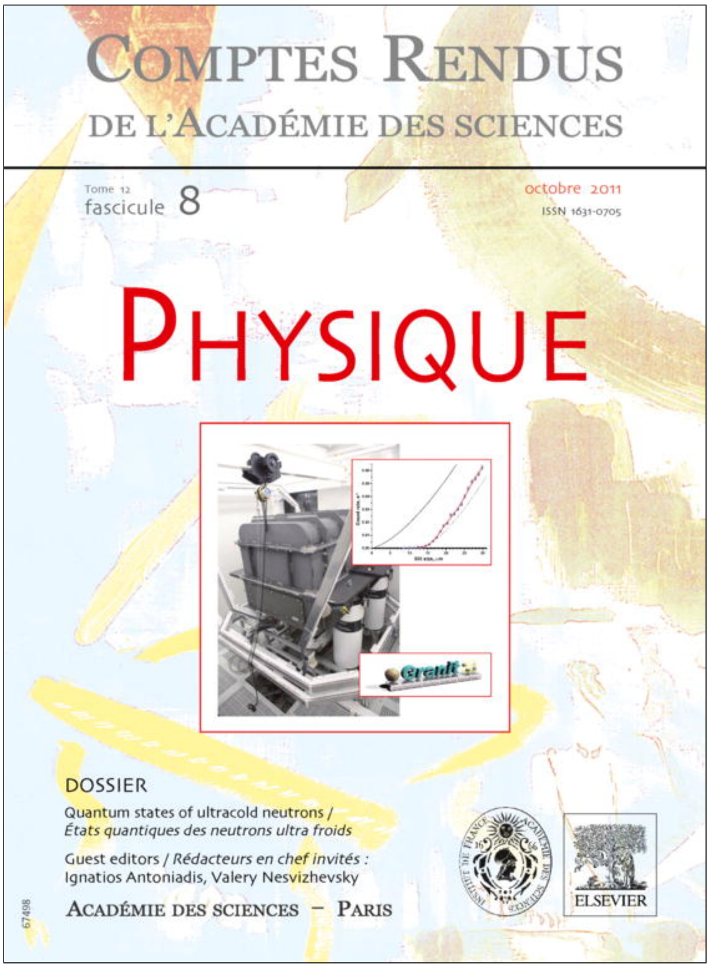

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Ignatios Antoniadis, Valery Nesvizhevsky

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Ultra cold neutron quantum states / États quantiques des neutrons ultra froids

Centrifugal quantum states of neutrons

États quantiques centrifuge de neutrons

V.V. Nesvizhevsky [a,*], A.Yu. Voronin [b]

[a] *ILL, 6 rue Jules Horowitz, Grenoble, F-38042, France*
[b] *Lebedev Institute, 53 Leninskii pr., Moscow, RU-119991, Russia*

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ABSTRACT

The "whispering gallery" effect has been known since ancient times for sound waves in air, later in water and more recently for a broad range of electromagnetic waves from the radiofrequency region, through visible light to X-rays. It consists of wave localization in the vicinity of concave surfaces. For matter waves, it would include a new feature: a massive particle would be settled in quantum states, with parameters depending on its mass. Here, we present the first observation of such an effect as well as constraints for short-range forces that could be estimated from this first observation.

RÉSUMÉ

L'effet « de galerie de chuchotement » est connu depuis l'antiquité pour les ondes sonores dans l'air, puis dans l'eau et, plus récemment, pour une large gamme d'ondes électromagnétiques, des radiofréquences, à la lumière et aux rayons X. Il consiste en la localisation d'ondes au voisinage de surfaces concaves. Pour les ondes de matière, il pourrait inclure une nouvelle particularité: une particule massive dans des états quantiques dont les paramètres dépendent de la masse. Ici, nous présentons la première observation d'un tel effet ainsi que les limites pour les forces à courte portée, qui pourraient être estimées à partir de cette première observation.

1. Introduction

Owing to the whispering gallery effect, sound can reach a person on the opposite side of a building, or even complete a circle, imitating an "echo". Lord Rayleigh explained and described quantitatively this phenomenon in his "Theory of sound" [1], thus giving birth to the science of acoustics. Whales are believed to communicate over long distances, profiting from a similar effect in surface layers of sea water. The electromagnetic whispering gallery waves from radio to light ("glory" or "heiligenschein") and X-ray frequencies are of ever-growing interest [2,3], owing to their multiple applications. Prior to the work described here [4], an analogous phenomenon has never been measured with matter waves. At certain condition, the problem can be reduced to a quantum particle above a mirror in a linear potential – the so-called quantum bouncer, in analogy with the neutron quantum motion in the Earth's gravitational field above a flat mirror [5]. On the other hand, an optical analogue of a quantum particle bouncing on a hard surface under the influence of gravity is experimentally

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[*] Corresponding author.
E-mail address: nesvizhevsky@ill.eu (V.V. Nesvizhevsky).

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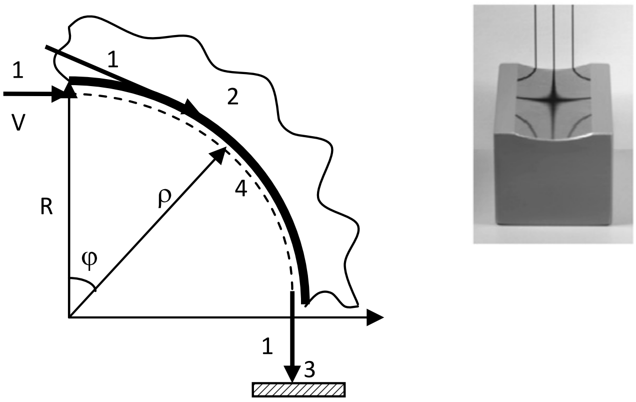

Fig. 1. A scheme of the neutron centrifugal experiment. 1: Classical trajectories of incoming and outgoing neutrons, 2: cylindrical mirror, 3: neutron detector, 4: quantum motion along the mirror surface. Insert: A photo of the single-crystal cylindrical silicon mirror used for the presented experiments, with an optical reflection of black stripes for illustrative purposes.

Fig. 1. Schéma de l'expérience de centrifugation des neutrons. 1 : trajectoires classiques des neutrons incident et sortants, 2 : miroir cylindrique, 3 : détecteur de neutrons, 4 : mouvement quantique le long de la surface du miroir. Dans l'insert : une photo du miroir cylindrique en silicium monocristallin utilisé pour les expériences présentées ici ; pour illustration, on voit la réflexion optique de lignes noires.

demonstrated using a circularly curved optical waveguide [6]. Spatially resolved tunneling optical microscopy measurements of multiple beam reflections at the waveguide edge clearly showed the appearance of wave packet collapses and revivals (either integer or fractional), corresponding to the full quantum regime of the quantum bouncer. Quantum revivals [7] might have applications to experiments with the GRANIT spectrometer [8,9], in particular to the neutron whispering gallery quantum states [10,11,4,12]. Two kinds of initial localization of neutrons could be considered: in position (easier to apply to gravitational quantum states [5,13–16]) and in energy (easier applied to centrifugal quantum states). Revivals are an extremely sensitive tool to study surface potentials, however, their extreme sensitivity to various experimental parameters might lead to various false effects, to be studied experimentally. The last two topics were presented at the GRANIT-2010 workshop, but not included in this article as the related material has been already published elsewhere.

In this article, the observation of centrifugal quantum states of neutrons is presented in Section 2, and a theoretical description of centrifugal quantum states of neutrons and their sensitivity to short-range interactions are given in Section 3.

2. Observation of centrifugal quantum states of neutrons (Valery Nesvizhevsky)

Consider scattering of a cold neutron by a perfect cylindrical mirror with a radius of a few centimeters as shown in Fig. 1.

The mirror is described by a uniform neutron-nuclear optical potential, which reflects (classically) neutrons with radial velocity components lower than the critical velocity corresponding to this potential. On the other hand, the neutron is affected by a huge centrifugal acceleration of the order of a million times the Earth's gravitational acceleration. The tangential neutron motion is essentially classical, while for the radial motion quantum effects are dominant. The quantum states are settled in a bounding well of nearly triangular shape formed by the centrifugal potential and the mirror potential. These are quasi-stationary states, as the probability of tunneling through the trapping potential barrier is never zero, although it could be negligible for deeply bound states.

The method to study the quantum states is based on a continuous variation of the bounding triangle barrier width. The width, the energy and the number of states depend strongly on the neutron velocity. In particular, the transmitted neutron flux increases sharply from zero when the neutron velocity (wavelength) approaches a characteristic cut-off value corresponding to the appearance of the lowest quasi-stationary state.

An alternative method for studying such centrifugal states consists of measuring the radial velocity distribution using a position-sensitive neutron detector, placed at a distance from the mirror. In particular, if a single long-living state is populated, we could measure directly the distribution of radial velocity components in this quantum state. Evidently, the most powerful method consists of combining the two options that is a simultaneous measurement of the scattering angle and the neutron velocity.

A typical scattering pattern is shown in Fig. 2.

Neutrons enter from the entrance edge of a truncated cylindrical mirror (Fig. 1). The "fingerprints" of the quantum states in Fig. 2 have a "V" shape. This can be understood as follows. The average deviation angle is equal to the angular mirror size; the "fingerprint" is centered on this value; thus, the radial velocity distribution is symmetric relative to the zero value. For classical consecutive Garland trajectories of neutrons, the width of the "V" letter would be exactly proportional to the neutron wavelength (Fig. 2). A manifestation of the observed centrifugal quasi-stationary states consists of the sharp cutoff in the neutron flux wavelength, corresponding to the appearance of the lowest quantum state. Another manifestation is the stripe structure inside the "V" shape. It is explained by interference of a few transmitted quantum states. A theoretical simu-

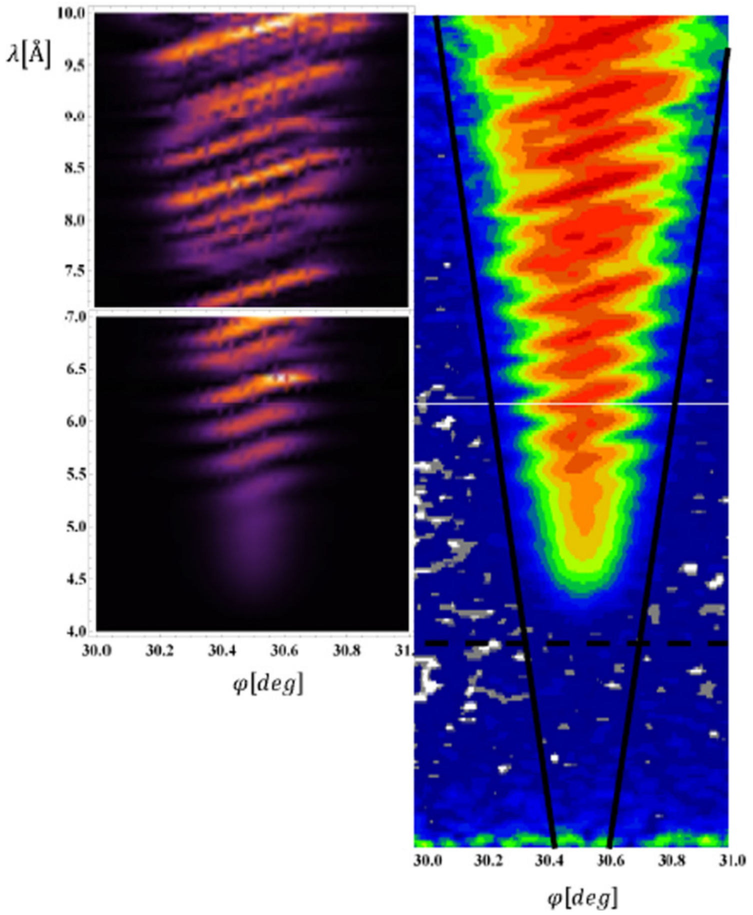

Fig. 2. On right: The scattering probability as a function of neutron wavelength (vertical axis) and deviation angle (horizontal axis). The geometrical angular size of the mirror is 30.5°. The inclined solid lines show the signal shape for the classical Garland trajectories. The dashed line illustrates a characteristic wavelength cutoff. On left: Theoretical simulation of the data.

Fig. 2. A droite : Probabilité de diffusion en fonction de la longueur d'onde des neutrons (axe vertical) et de l'angle de déviation (axe horizontal). La dimension angulaire géométrique du miroir est de 30.5°. La ligne continue inclinée représente la forme du signal pour des trajectoires de Garland classiques. La ligne discontinue représente une longueur d'onde de coupure caractéristique. Sur la gauche : Simulation théorique des données.

lation of the data, shown in Fig. 2, reproduces the measurement in detail. Some of the difference between the experimental data and the simulation is probably due to the thin oxide layer on the mirror surface.

At certain conditions, the problem can be reduced to a quantum particle above a mirror in a linear potential – the so-called quantum bouncer, in analogy with the neutron quantum motion in the Earth's gravitational field above a flat mirror [5]. These two phenomena provide a first direct demonstration of the weak equivalence principle for an object in a quantum state: although the independence of a free fall on mass does not hold in the quantum limit, quantum states of a massive body in a locally uniform gravitational field and those in a system moving with equal acceleration are equivalent. Both problems, the centrifugal and gravitational ones, provide an excellent laboratory for studying neutron quantum optics phenomena. Deeply bound whispering-gallery states are long-living and weakly sensitive to surface potentials thus could be used for searches for extra short-range forces [17–19]; highly excited states are short-living and very sensitive to the wall potential shape. Therefore, they are a promising tool for studying fundamental neutron–matter interactions, quantum neutron optics and surface physics.

3. Theoretical description of centrifugal quantum states of neutrons and their sensitivity to short-range interactions (Alexey Voronin)

The problem of neutron scattering on a cylindrical mirror can be solved in a closed form, free from fitting phenomenological parameters [10,12]. This clear mathematical background makes the problem attractive for studying physical effects, responsible for deviations from the benchmark theory of neutron scattering on an ideal cylinder, which is characterized by the neutron-optical potential. Such additional effects include neutron scattering by hypothetical extra-forces, suggested in extensions of the Standard Model. So far the constraints on the strength and the characteristic range of such extra-forces could be derived by comparing experimental data to theoretical predictions. The main result of our theoretical study of the whispering gallery scattering of neutrons on a cylindrical mirror consists in establishing a relation between the centrifugal

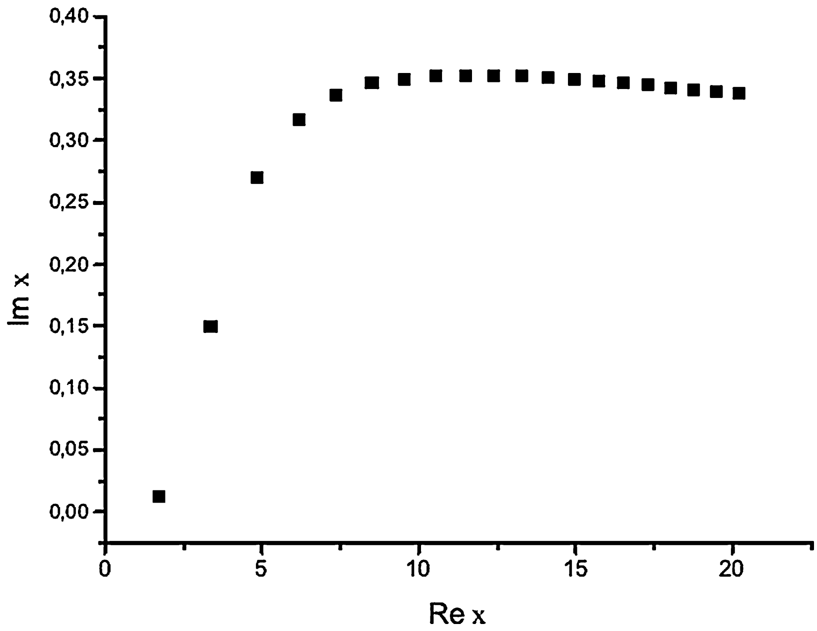

Fig. 3. Values of the S-matrix poles in the first quadrant of the complex momentum plane.

Fig. 3. Valeurs des pôles de la matrice S dans le premier cadran du plan complexe des impulsions.

quasi-stationary states of neutrons, localized due to superposition of centrifugal and optical potentials, and the neutron scattering amplitude. This relation is given by the following equation:

$$f(\varphi) = -2i\left(\frac{\mu_0}{2}\right)^{1/3} \exp(i\mu_0\varphi)\sqrt{\frac{2\pi\hbar}{p}}\sum_{n=1}^{\infty}\text{Res}\, S(\lambda_n)\exp\left(-i\lambda_n\left(\frac{\mu_0}{2}\right)^{2/3}\varphi\right) \tag{1}$$

Here $\mu_0 = \frac{MvR}{\hbar}$, M is the neutron mass, v is the neutron velocity, R is the radius of a cylindrical mirror, $p = Mv$, S is the scattering S-matrix and λ_n is the pole of the S-matrix in the complex momentum plane [10,12].

The physical sense of the above expression is transparent. For large scattering angles (which we are interested in) the main contribution to the scattering amplitude is given by the S-matrix poles with the smallest imaginary parts (Fig. 3). These poles correspond to the narrow quasi-stationary centrifugal states mentioned above. The energy and width of such states can be exactly calculated. Indeed, such states are the well-known solution of the Schrödinger equation in the linear potential with the emission boundary condition:

$$\chi \sim \begin{cases} Bi(u_0 - x - \lambda_n) + iAi(u_0 - x - \lambda_n), & z \geqslant 0 \\ Ai(-x - \lambda_n), & z < 0 \end{cases} \tag{2}$$

where $u_0 = U(\frac{2R^2}{\hbar^2 M v^4})^{1/3}$, $x = z(\frac{2M^2v^2}{\hbar^2 R})^{1/3}$, U is the depth of optical potential.

Theoretically predicted results are found to be in a close agreement with the experimental data. This agreement could be used for establishing limits on strength-range of extra-forces; they are shown as the line 6 in Fig. 1 in Ref. [17]. The accuracy of presented constrain is limited by our knowledge of possible systematic effects. The accuracy of measurements of this kind could be largely improved in further dedicated experiments.

4. Conclusion

We described the new phenomenon of the neutron whispering gallery phenomenon and a theoretical formalism to describe it. It is a promising tool for studying fundamental neutron–matter interactions, quantum neutron optics and surface physics effects. In particular, we estimate that it provides the best neutron method for constraining extra short-range forces in a broad distance range, with large potential for further improvement in sensitivity.

Acknowledgements

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