Conversion of Cyclops octagonal images to cylindrical images (July 2012)

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The conversion of CYCLOPS raw images to cylindrical detector is explained shortly in the present document. The meaning of the different symbols can be deduced simply from the figures.

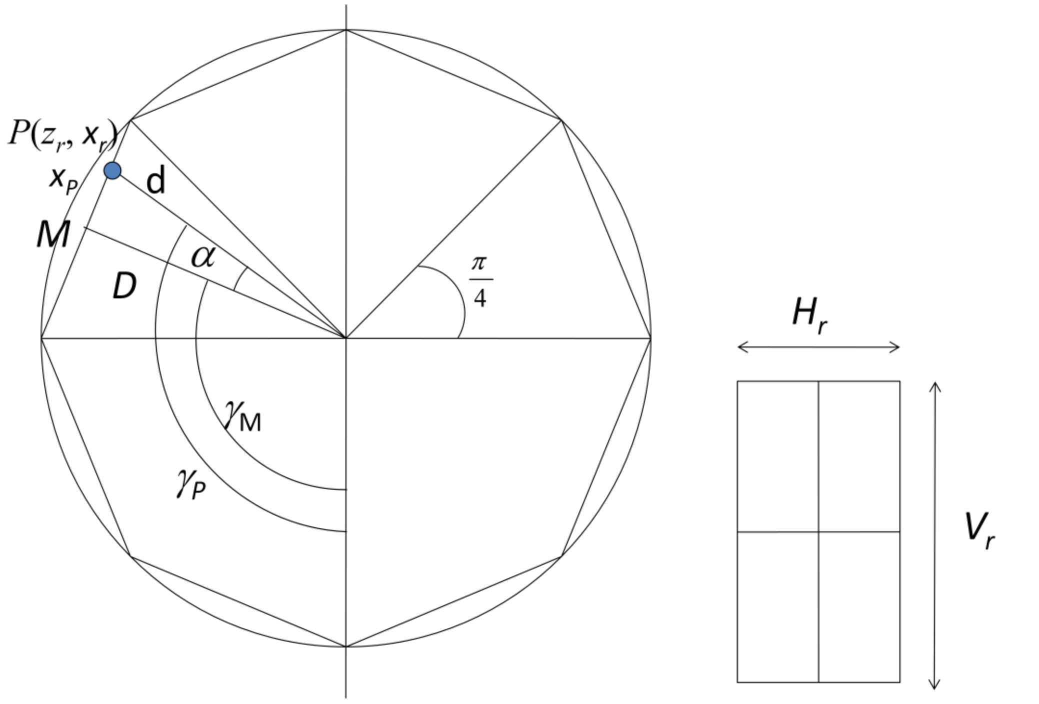

Figure1: Positioning on an arbitrary point in the octagonal-rectangular image and its relation to the angular coordinates. The composite detector is formed by 8 rectangular detectors of dimensions H_r and V_r.

The basic geometrical characteristics of the conversion are summarised in Figures 1 and 2. The point P has coordinates (z_r, x_r) in the rectangular composite detector with polar angles $(\gamma, \nu)_P$. The polar angles should be the same in the projected cylindrical image. In the perfect octagonal detector one has the relation:

$$\tan\frac{\pi}{8} = \frac{H_r}{2D} \rightarrow H_r = 2D\tan\frac{\pi}{8}$$

The deployed composite detector is represented in Figure 2, in which we see the relation between the discrete matrix storing the image and the metrical features of the detector. The value of x_M depends on the detector number, n_d, whereas z_M is fixed.

The image is represented by a matrix of dimension $N_{row} \times N_{col} = n_{vr} \times n_{hr}$, so there is the following correspondence:

$(z_r, x_r) \rightarrow (i_r, j_r)$: $z_r = (i_r - 1)p_z$; $\quad x_r = (j_r - 1)p_x$; $\quad z_M = \dfrac{V_r}{2} = \dfrac{1}{2}(n_{vr} - 1)p_z$

Where the pixel sizes in the rectangular composite detector along x and z are given by:

$$p_x = \frac{H_T}{n_{hr} - 1} = \frac{8H_r}{n_{hr} - 1}; \quad p_z = \frac{V_r}{n_{vr} - 1}$$

We have to calculate the coordinates (Z_c, X_c) in the deployed cylindrical surface in terms of the rectangular coordinates (z_r, x_r) with the constraint that both have the same polar angles. For a cylindrical surface of radius R_c we have the following relations:

$$x_c = \gamma R_c \qquad\qquad X_c = \gamma R_c$$
$$z_c = R_c \tan\nu \qquad\qquad Z_c = z_M - R_c \tan\nu$$

On the left we have the usual relations when taking the origin in the centre of the surface. On the right we have the coordinates when we use the same coordinate system as in the deployed rectangular surface (origin on the top left part, Figure 2).

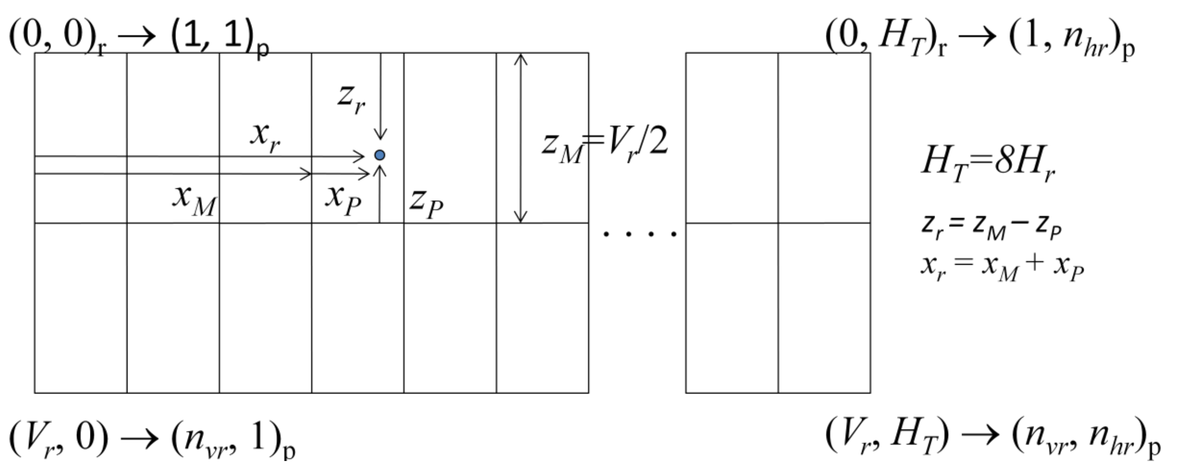

Figure 2: Composite rectangular detector.

We can calculate the number of the detector n_d from the x-coordinate in the rectangular composite detector as:

$$n_d = \mathrm{int}\frac{x_r}{H_r} + 1 \approx \mathrm{int}\frac{(j_r - 1)p_x}{n_{hr}/8p_x} + 1 = \mathrm{int}\left[\frac{(j_r - 1)}{n_H}\right] + 1$$

From the figures we can obtain applying elementary geometry the following set of relations:

$$\gamma_P = \gamma_M^{(n_d)} + \alpha \qquad \gamma_M^{(n_d)} = (n_d - 1)\frac{\pi}{4} + \frac{\pi}{8} = (2n_d - 1)\frac{\pi}{8}$$

$$x_P = x_r - x_M^{(n_d)} \qquad x_M^{(n_d)} = (n_d - 1)H_r + \frac{H_r}{2} = (2n_d - 1)\frac{H_r}{2}$$

$$x_P = x_r - (2n_d - 1)\frac{H_r}{2}; \qquad \gamma_P = (2n_d - 1)\frac{\pi}{8} + \tan^{-1}\frac{x_P}{D}$$

$$\tan\nu = \frac{z_M - z_r}{d}; \quad d = \sqrt{D^2 + x_P^2}$$

$$X_c = \gamma R_c = R_c\left[(2n_d - 1)\frac{\pi}{8} + \tan^{-1}\frac{x_P}{D}\right] = R_c\left[(2n_d - 1)\frac{\pi}{8} + \tan^{-1}\frac{2x_r - (2n_d - 1)H_r}{2D}\right]$$

$$Z_c = z_M - R_c \tan\nu = z_M - R_c\frac{z_M - z_r}{\sqrt{D^2 + x_P^2}}$$

Using explicitly the pixels, one can obtain the relation between the indices of the cylindrical (i_c, j_c) and rectangular (i_r, j_r) images. We call c_z and c_x the pixel sizes in the projected cylindrical image that we can select as we want in terms of the cylindrical radius R_c and dimensions of the final image: $n_{vc} \times n_{hc}$. The deduction of the final expressions relating the indices (i_c, j_c) =F(i_r, j_r) is done below:

$$Z_c = (i_c - 1)c_z \qquad i_c = 1,\dots n_{vc}; \qquad X_c = (j_c - 1)c_x \qquad j_c = 1,\dots n_{hc}$$

$$\tan\nu = \frac{z_M - z_r}{\sqrt{D^2 + x_P^2}} = \frac{(n_{vr}-1)p_z/2 - (i_r - 1)p_z}{p_x\sqrt{D_{(p)}^2 + x_{P(p)}^2}} = \frac{p_z}{2p_x}\frac{(n_{vr}-1) - 2(i_r-1)}{\sqrt{D_{(p)}^2 + x_{P(p)}^2}}$$

$$\gamma = (2n_d - 1)\frac{\pi}{8} + \tan^{-1}\frac{2x_r - (2n_d - 1)H_r}{2D} = (2n_d - 1)\frac{\pi}{8} + \tan^{-1}\left\{\frac{(j_r - 1)}{D_{(p)}} - (2n_d - 1)\tan\frac{\pi}{8}\right\}$$

$$D = D_{(p)}p_x; \quad x_P = x_{P(p)}p_x; \quad H_r = H_{r(p)}p_x; \quad p_x = \frac{8H_r}{n_{hr} - 1}; \quad H_r = 2D\tan\frac{\pi}{8}; \quad D_{(p)} = \frac{n_{hr} - 1}{16\tan\frac{\pi}{8}}$$

$$x_P = x_{P(p)}p_x = x_r - (2n_d - 1)\frac{H_r}{2} = (j_r - 1)p_x - (2n_d - 1)\frac{n_{hr} - 1}{16}p_x$$

$$x_{P(p)} = j_r - 1 - (2n_d - 1)\frac{n_{hr} - 1}{16}$$

$$(j_c - 1)c_x = \gamma R_c; \quad \frac{R_c}{c_x} = \frac{n_{hc} - 1}{2\pi}$$

$$j_c = \frac{n_{hc} - 1}{2\pi}\left[(2n_d - 1)\frac{\pi}{8} + \tan^{-1}\left\{\frac{(j_r - 1)}{D_{(p)}} - (2n_d - 1)\tan\frac{\pi}{8}\right\}\right] + 1$$

$$(i_c - 1)c_z = \frac{1}{2}(n_{vr} - 1)p_z - R_c\frac{p_z}{2p_x}\frac{(n_{vr} - 2i_r + 1)}{\sqrt{D_{(p)}^2 + x_{P(p)}^2}}$$

$$i_c = \frac{(n_{vr} - 1)p_z}{2c_z} - \frac{c_x p_z}{4\pi c_z p_x}\frac{(n_{vr} - 2i_r + 1)(n_{hc} - 1)}{\sqrt{D_{(p)}^2 + x_{P(p)}^2}} + 1$$

Using the explicit formulae we can ask for a conversion to an arbitrary cylinder detector. We have to change the input instrument characteristics. The octagonal detector is characterized by the total length (in mm) of the rectangular image H_T=8H_r, the vertical length V_r, and the corresponding number of pixels (n_{hr}, n_{vr}). The distance D is deduced from H_r. The practical formulae to calculate the pixel on the cylindrical image from which the user should specify the radius (R_c) the height (V_c) and the number of horizontal and vertical pixels (n_{hc}, n_{vc}) are emphasised below. We first provide the direct relation (i_c, j_c) =F(i_r, j_r), for that we need to do some previous calculations as shown above.

The number of the current detector is: $n_d = \mathrm{int}\left[\frac{(j_r - 1)}{n_H}\right] + 1$

The local coordinate, w.r.t. the middle position, in pixels is: $x_{P(p)} = j_r - 1 - (2n_d - 1)\frac{n_{hr} - 1}{16}$

The distance from the centre of a rectangular detector to the centre of the octagon, expressed in pixels is: $D_{(p)} = \dfrac{n_{hr}-1}{16\tan\dfrac{\pi}{8}}$

The final (i_c, j_c) =F(i_r, j_r) relations are:

$$j_c = \frac{n_{hc}-1}{2\pi}\left[(2n_d-1)\frac{\pi}{8}+\tan^{-1}\frac{x_{P(p)}}{D_{(p)}}\right]+1$$

Notice that this expression doesn't contain the pixel size or any real dimension of the detector. Only number of pixels and angular relations occur. This is not the case for the i_c index.

$$i_c = \frac{(n_{vr}-1)p_z}{2c_z}\left[1-\frac{c_x(n_{hc}-1)}{2\pi p_x(n_{vr}-1)}\frac{(n_{vr}-2i_r+1)}{\sqrt{D_{(p)}^2+x_{P(p)}^2}}\right]+1$$

The pixel sizes enter directly in this formula. Only for particular values of the cylinder radius and number of pixels, in both horizontal and vertical directions, these values may disappear (e.g. c_z=p_z and c_x=p_x). The same formula in terms of real dimensions of the detectors can be written as:

$$i_c = \frac{V_r}{2V_c}(n_{vc}-1)\left[1-\frac{R_c}{H_T}\frac{(n_{hr}-1)}{(n_{vr}-1)}\frac{(n_{vr}-2i_r+1)}{\sqrt{D_{(p)}^2+x_{P(p)}^2}}\right]+1 = f_v\left[1-f_h\frac{(n_{vr}-1)-2(i_r-1)}{\sqrt{D_{(p)}^2+x_{P(p)}^2}}\right]+1$$

The above expressions correspond to the direct calculation of the projected pixels in the cylindrical image. The drawback of this approach in practical calculation is that going through all the pixels in the rectangular image we are not sure that we will fill up the complete image matrix representing the cylindrical image due to the eventual differences in the number of pixels and dimensions.

Let us calculate the inverse relations (i_r, j_r) =F^{-1}(i_c, j_c) which are much more convenient for calculation of the final image. First we have to calculate the number of the detector in a different way as a function of j_c. We obtain easily the following relations:

$$x_c = \gamma R_c = (j_c-1)c_x \to \gamma = (j_c-1)\frac{c_x}{R_c} = (j_c-1)\frac{2\pi}{n_{hc}-1}; \qquad \frac{R_c}{c_x} = \frac{n_{hc}-1}{2\pi}$$

$$n_d = \operatorname{int}\left[\frac{(j_r-1)}{n_H}\right]+1 = \operatorname{int}\frac{4\gamma}{\pi}+1 = \operatorname{int}\frac{4}{\pi}(j_c-1)\frac{2\pi}{n_{hc}-1}+1 \simeq \operatorname{int}\frac{(j_c-1)}{n_{hc}/8}+1$$

So the final relations are:

$$j_r = D_{(p)}\tan\left[\frac{2\pi(j_c-1)}{n_{hc}-1}-(2n_d-1)\frac{\pi}{8}\right]+(2n_d-1)\frac{n_{hr}-1}{16}+1$$

$$i_r = \frac{1}{2}\left\{(n_{vr}-1)-\left[1-\frac{i_c-1}{f_v}\right]\frac{\sqrt{D_{(p)}^2+x_{P(p)}^2}}{f_h}\right\}+1$$

Final relations giving the pixels in the original image from the pixels in the projected image.