Beginning with the first experiments, all observations have shown that the UCN confinement time is much shorter than the theoretical value. The maximum attainable number of successive collisions of a UCN with a beryllium trap wall turned out to be 100 times (and for a solid oxygen trap wall 1000 times) smaller than the calculated one. The nature of this discrepancy has not been fully understood [24]. It is likely that the anomalous UCN losses are related to the recently observed quasi-elastic UCN scattering from trap walls of different materials with a probability of $\sim 10^{-7}$ per impact, whereby the energy of the UCN increases nearly twofold [25, 26]. Also observed, with a probability an order of magnitude lower, were processes of further cooling of UCNs at a wall whose temperature (300 K) is six orders of magnitude higher than the temperature corresponding to the UCN temperature (10^{-3} K). Fundamentally, such processes are not forbidden, but their intensity proved to be 5–6 orders of magnitude higher than expected. Experiment has shown that the observed phenomenon is caused by inelastic UCN scattering by very small particles with an atomic mass of $\sim 10^7$ weakly coupled to the surface, which are in the state of thermal motion determined by the wall temperature. These nanoparticles can be discerned by modern microscopy techniques. However, investigation of their dynamics seems to be inaccessible for other methods except the UCN technique.

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Investigation of quantum neutron states in the terrestrial gravitational field above a mirror

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1. Introduction

Quantum mechanics implies that an elementary particle or any other material object resides in bound quantum states in a sufficiently deep potential well irrespective of the nature of the attractive potential. This means that the set of allowed energy levels is defined by the particle mass and the shape of the attractive potential, while the probability that the particle is at an arbitrary point in space is equal to the squared modulus of its wave function in the corresponding quantum state at this point. And the particle-retaining potential may be any of the four known kinds: electromagnetic, strong, weak, or gravitational. A natural example of quantum states of matter is the states of electrons in atoms [1] and, in a strong nuclear field, the states of neutrons and protons in nuclei [2]. The observation of a similar phenomenon in the gravitational field is complicated by the extreme weakness of gravitational interaction in laboratory conditions. However, it is nevertheless possible when the terrestrial gravitational field makes up one of the walls of a potential well and the other wall is a horizontal mirror that reflects the particle and thereby bounds the domain in which it can move. The Schrödinger equation that describes the quantum states of a particle of mass m moving in the terrestrial gravitational field with the acceleration of gravity g above a perfect horizontal mirror has been analytically solved in textbooks on quantum mechanics [3–7] and theoretically investigated in many papers, for instance in Refs [8–10]. For macroscopic objects under ordinary conditions, the corresponding quantum effects are negligible, but they may be significant for bodies with a small mass, e.g., for elementary particles [11], in particular, for neutrons [12, 13] or atoms [14, 15]. The dominant interaction for charged particles is the electromagnetic one, which makes observing their quantum states in the gravitational field practically unfeasible. Experiments with neutrons and neutral atoms are in principle possible, even though they are hampered by many methodical factors like, for instance, the extremely low phase-space density of neutrons with sufficiently low energy (the effective neutron temperature in the lower quantum state is ~ 20 nK !) or the difficulties associated with developing a perfect mirror for neutral atoms. The technique of measuring the quantum states of neutral atoms in a gravitational field was elaborated, for instance, in Refs [14–18]. As a mirror for atoms, Refs [14 – 18] employed a standing laser wave [19 – 21] decaying exponentially at the interface between two media or the gradient of the magnetic field induced by periodically arranged conductors, each two neighboring conductors carrying currents in the opposite directions [18]. The scheme of an experiment with neutrons was proposed in Refs [12, 13]. The idea of this experiment [18] was further developed when a precision gravitational neutron spectrometer [22] was made at the Institut Laue – Langevin. The first spectrometer-based measurement made it possible to discover the lowest stationary quantum state of neutrons in the terrestrial gravitational field [24 – 26] above a mirror. In this experiment, measurements were made of the transmittance for neutrons of a narrow gap between a horizontal mirror below and a scatterer/absorber above (for brevity, henceforward it will be referred to simply as a scatterer). This gap was opaque for neutrons when its width was smaller than the characteristic dimension of the neutron wave function in the lowest quantum state. This work was the first experimental observation of the quantum state of matter in a potential well formed by the gravitational field. The kindred phenomenon of neutron phase shift induced by the terrestrial gravitational field was measured by Colella et al. [27] with the use of a neutron interferometer. Other schemes for observing quantum effects related to neutron motion in the gravitational field were discussed, for instance, in Refs [28 – 30]. From the methodical standpoint, the most convenient portion of the neutron spectrum in these experiments usually are ultracold neutrons (UCNs) having a velocity of ~ 5 m s^{-1} and an energy of $\sim 10^{-7}$ eV, the softest portion of the neutron spectrum of cold neutron sources. UCNs have been extensively employed in basic physics since the first experiments on their storage in traps carried out in Dubna [31], which has brought forth significant progress in this area during the past 35 years, including the methods of their spectrometry and detection, as well as the knowledge of the properties of their interaction with matter. In a significant part of the works with UCNs, use was made of the UCN channel [32] of the high-flux reactor of the Institut Laue – Langevin in Grenoble. The properties of UCNs and their application areas are represented in numerous publications, for instance in Refs [33 – 35].

The present report outlines an experiment aimed at studying the effect of neutron state quantization in the terrestrial gravitational field above a horizontal mirror, which was measured earlier in Refs [24 – 26]. The full and comprehensive account of the findings from this experiment and their analysis will be published later [36]. Here, we only list the main objectives of this experiment and ways to solve them and enlarge on only one method [22]. This method involves direct measurement of the spatial distribution of the neutron density in quantum states in the terrestrial gravitational field above a mirror with the aid of a position-sensitive detector with a high spatial resolution [23].

2. General experimental scheme

The experimental setup is shown in Fig. 1. For a neutron detector, advantage was taken of either a low-background neutron counter, like in Refs [24, 25], or a position-sensitive detector with a spatial resolution of 1 – 2 μm [23]. In the former case, the experiment involved measurements of the neutron flux passing through the mirror – scatterer gap as a function of the gap width, which could be smoothly varied and precisely measured. In the latter case, the spatial distribution of the likelihood of detecting a neutron in the quantum states above the mirror was directly measured employing the position-sensitive detector.

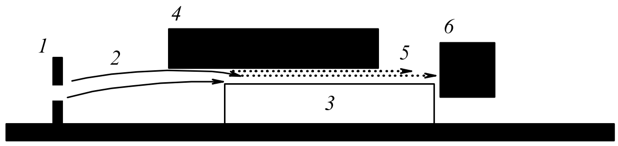

Figure 1. Basic schematic of the experiment. From left to right: the vertical solid line sections show the upper and lower plates of the inlet collimator (*1*); the solid arrows correspond to the classical parabolic neutron trajectories (*2*) in the interval between the collimator and the entry to the gap between the mirror (*3*, the empty rectangle below) and the scatterer (*4*, the black rectangle above). The dashed horizontal arrows illustrate the quantum neutron motion above the mirror (*5*), and the black square is the neutron detector (*6*). The mirror – scatterer gap width can be varied and measured.

The neutron flux at the point of entry into the experimental facility (at the left of Fig. 1) is uniform in height and angle-isotropic over ranges which exceed by a order of magnitude the gap size and the angular acceptance of the spectrometer. The spectrum of the horizontal neutron velocity component is preset with the aid of an input collimator; both its plates can be placed at a requisite height independently of each other. The detector measures the neutron flux at the spectrometer output.

In the ideal case, the horizontal and vertical neutron motions are independent. This is true when the reflection of neutrons from the mirror does not mix their horizontal and vertical velocity components, the effect of the scatterer on the motion of the neutrons penetrating the gap can be ignored, and the neutrons experience only the terrestrial gravitational field and the reflective potential of the mirror, while the effects of the remaining forces are negligible. In this case, the horizontal neutron motion obeys classical laws (with an average velocity of ~ 5 m s^{-1}), while the vertical motion is quantized, with the vertical velocity component equal to several centimeters per second and the energy of the vertical motion equal to several peV (10^{12} eV). The extent to which each of these conditions is valid is not evident *a priori* and should be verified experimentally.

In the quasiclassical approximation [3 – 7], which is highly accurate in the problem under consideration, in accordance with the Bohr – Sommerfeld formula the neutron energy in the quantum states E_n ($n = 1, 2, 3, \ldots$) is

$$E_n \cong \sqrt[3]{\left(\frac{9m}{8}\right)\left[\pi\hbar g\left(n - \frac{1}{4}\right)\right]^2}\,. \qquad (1)$$

One can see from formula (1) that the energy value depends only on the neutron mass m, the gravitational acceleration g, and the Planck constant $\hbar = 6.6 \times 10^{-16}$ eV, and is independent of the parameters of the mirror, which is an infinitely high and steep potential step relative to the characteristic parameters of the problem. For comparison, the neutron energy in the lower quantum state ($\sim 10^{-12}$ eV) is much less than the Fermi potential of the mirror ($\sim 10^{-7}$ eV) and the Fermi potential buildup region (~ 1 nm) is much shorter than the neutron wavelength in the lower quantum state (~ 10 μm). The rigorous solution of the corresponding Schrödinger equation is given, for instance, in Refs [3 – 7].

The exact expression for the energy values E_n is, unlike formula (1), written with the use of the special Airy functions and is therefore more cumbersome. However, it possesses the same property: it depends only on m, g, $\hbar$, and n. The energy levels E_n correspond to the solutions of the following equation:

$$\mathrm{Ai}\left(-\frac{\sqrt[3]{2}}{\sqrt[3]{mg^2\hbar^2}}\,E\right)=0\,, \tag{2}$$

where Ai(...) is the Airy function.

The neutron eigenfunction $\psi_n(z)$ in the nth quantum state at a height z is [3–7]

$$\psi_n(z)=C_n\,\mathrm{Ai}\left(\frac{z}{z_0}-\alpha_n\right), \tag{3}$$

where $z_0=\sqrt[3]{\hbar^2/2m^2g}=5.87$ μm is the characteristic scale length of the quantum states, α_n are the roots of the Airy function $\mathrm{Ai}(-\alpha)$, and the coefficient $C_n=\left(z_0\int_{-\alpha_n}^{\infty}\mathrm{Ai}(\zeta)^2\,\mathrm{d}\zeta\right)^{-1/2}$.

In both cases considered above — in the measurement of the total neutron flux through the mirror–scatterer gap and in the direct measurement of the spatial neutron density distribution — we have to deal with the wave functions of neutrons in quantum states rather than with the energies of these quantum states.

3. Features and goals of the experiment

The main goals of this experiment were to make sure that no systematic errors were possible as well to study the areas of application of both the effect of neutron quantization in the gravitational field above a mirror and the ultrahigh-resolution picoelectron-volt neutron spectrometry technique under development. Both the experimental facility and the measurement technique were similar to those employed previously [24–26]. That is why we only mention the main distinctions of the last measurement, while the details of the experiment itself can be found in previous publications.

In the first experiment [24, 25], the accuracy of positioning the mechanical scatterer above the mirror was adequate for the problem being solved, for it enabled one to register the neutron lower quantum state in the potential well formed by the terrestrial gravitational field and the mirror. However, further improvement of the instrumental resolution of the spectrometer and increasing the measurement accuracy called for its development. In the present experiment, the mirror–scatterer gap was measured by a more precise capacitive technique. Several metallic electrodes were deposited on the plane glass surface of the lower mirror. The continuous metallic coating of the lower rough surface of the scatterer fulfilled the function of the upper electrode. The electrode separation was strictly, to within several angstroms, related to the magnitude of the capacitance made up by the electrodes. This ensures a high repeatability of measurements and the absence of so-called differential errors when setting the separation (the capacitance is a monotonic smooth function of the separation). To eliminate the so-called integral errors, primarily the possible shift in the determination of the zero scatterer–mirror gap, we develop techniques for independent distance calibration. The new, higher-precision measurement of the mirror–scatterer gap transmittance for neutrons as a function of the gap width confirmed the conclusions of Refs [24, 25] about the existence of the neutron lower quantum state in the terrestrial gravitational field above the mirror and did not reveal noticeable systematic errors related to the positioning system in the first experiment.

We considered, both theoretically and experimentally, the factors which determine the resolution of our spectrometer. Several models were proposed to describe the dependence of the detector counting rate on the mirror–scatterer gap width:

1. A model assuming that the loss of a neutron in the scatterer results from its tunneling from the classically allowed region above the mirror into the scatterer through the gravitational potential barrier.

2. A model which treats the scatterer as a complex potential with a large imaginary part responsible for the neutron loss.

3. A model with unperturbed neutron wave functions, where the neutron destruction probability in a quantum state is proportional to the probability that the neutron is located at the height of the scatterer.

4. A model assuming quantum-mechanical solutions for the neutron wave functions in quantum states and a classical expression for the likelihood of non-specular neutron reflection from the rough absorber.

More subtle and exact theoretical descriptions are being elaborated. For a seemingly significant difference of these models they make almost coincident predictions for the dependence of the mirror–scatterer gap transmittance for neutrons on the gap width. The point is that the scatterer efficiency is high enough to destroy the quantum states almost without perturbing the corresponding wave functions, which are precisely known and are determined only by the terrestrial gravitational field and the mirror. Moreover, when the scatterer efficiency is high enough, the mirror–scatterer gap transmittance for neutrons depends only slightly on the exact efficiency value.

Therefore, both from the viewpoint of reliability and exactness of the theoretical description of the problem and from the viewpoint of improving the resolution of our spectrometer, the problem reduces to the increase of the scatterer efficiency. To this end we compared different types of scatterers and absorbers. Measurements revealed that their absorbing properties are of little consequence: their principle of operation relies primarily on the scattering of neutrons in non-specular directions (and their subsequent loss, because the frequency of their collisions with the mirror and the absorber increases many-fold). If in this case the amplitude of scatterer roughness is comparable to or greater than the wavelength of neutrons in the quantum state (several micrometers), the scatterer efficiency is high and approaches unity in the quasiclassical approximation.

Furthermore, we studied the quality of mirrors and estimated the storage time of neutrons in quantum states. This is of consequence for the study of the feasibility of applying the 'storage' technique, which involves the confinement of neutrons in a sealed mirror trap and their resonance transition from one quantum state to another. This can be attained, for instance, by the method of reflecting surface oscillations, by analogy with Ref. [37], or even through the variation of the gravitational field by placing a vibrating mass near the experimental facility. In the experiment conducted, the time of neutron storage in the quantum state amounted to at least 10–100 characteristic quantum-mechanical time intervals $\hbar/E_1=0.5$ ms, where $E_1=1.4$ peV is the energy of the lower quantum state. This means that the storage time is long enough for the simplest experiments with quantum states but should be significantly increased for precision measurements.

Such a precision measurement could be used, for instance, to verify the electrical neutrality of the neutron; for the verification of the weak equivalence principle, as applied to the motion of a quantum particle [38, 39]; or in the search for hypothetical additional short-range fundamental interactions [40 – 45], whose existence would lead to the violation of the weak equivalence principle. Should a strong enough additional interaction exist between the neutron and the mirror, the quantum state parameters would be deformed: the energies would shift and the wave functions would be distorted. The very fact that the neutron quantum states in the terrestrial gravitational field above the mirror exist, i.e., the absence of distortions, comparable to the gravitational potential, of the potential in which the neutron is embedded, allows us to bound the intensity of the additional short-range potential by a characteristic distance of 1 nm – μm. In this case, over a characteristic range of several nanometers this limitation is somewhat better than that obtained in other experiments. A consistent analysis of this problem and possibilities for its further improvement are presented in Ref. [46].

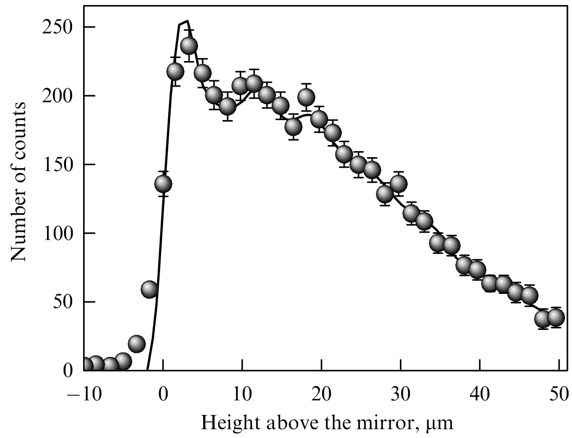

Figure 2. Preliminary results of measuring the probability of neutron residence above the mirror in the gravitational field obtained with the use of a nuclear track plastic detector with a thin uranium coating, which possessed a high spatial resolution. Plotted as the abscissa is the height above the mirror z in micrometers (the zero height was fitted from experimental data). Plotted as the ordinate is the number of events in the corresponding height interval. The solid line is the theoretical expectation assuming a perfect spatial resolution of the detector.

4. Measurement of the probability of detecting a neutron above the mirror with the aid of a position-sensitive detector

One of the ways of developing this experiment implies the use of position-sensitive detectors with a high spatial resolution, which are elaborated specifically for this problem.

Direct measurement of the spatial density distribution in a standing neutron wave is preferred over its investigation with the aid of a scatterer movable in height. The former technique is differential, for it permits the simultaneous measurement of the probability of neutron residence at all heights of interest. The latter technique is integral, since the information on the probability that neutrons reside at some height is in fact obtained by the subtraction of the values of neutron fluxes measured for two close values of the scatterer height. Clearly, the differential technique is much more sensitive than the integral one and makes it possible to gain the desired statistical accuracy much faster. This is of prime importance considering the extremely low counting rate in this experiment, even with the use of the highest UCN flux available today. Furthermore, the scatterer employed in the integral technique inevitably distorts the measured quantum states by deforming their eigenfunctions and shifting their energy values. The finite accuracy of taking these distortions into account results in systematic errors and ultimately limits the attainable accuracy of measurement of the parameters of quantum states. For these and other reasons, the employment of a position-sensitive detector to directly measure the probability of neutron residence above the mirror is highly attractive. However, no neutron detectors with a spatial

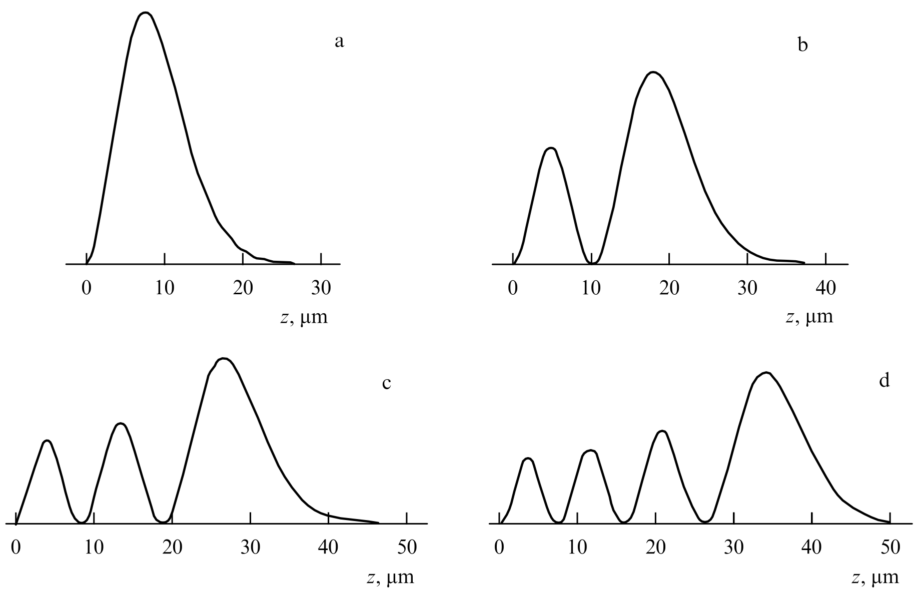

Figure 3. Neutron residence probability as a function of height above the mirror z for the 1st (a), 2nd (b), 3rd (c), and 4th (d) quantum states.

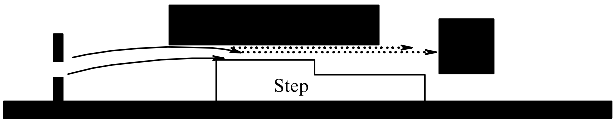

Figure 4. Basic diagram of the experiment with a small negative step on the lower mirror (compare with Fig. 1), which allows the transition of neutrons to higher quantum states (to the region to the right of the step).

resolution of ~ 1 μm required for this experiment existed. It was therefore necessary to develop such a detector and measuring technique: a plastic track nuclear detector (CR39) with a thin uranium coating ($^{235}UF_4$) described in Ref. [23]. The tracks arising from the entry into the plastic detector of one of the nuclei produced in the neutron-induced fission of the ^{235}U nucleus were increased to ~ 1 μm in diameter by way of chemical plastic development in an alkaline solution. The developed detector surface was scanned with an optical microscope over a length of several centimeters with an accuracy of ~ 1 μm. The sensitive ^{235}U layer is thin enough (< 1 μm) for the coordinates of neutron entry into the uranium layer to practically coincide with the coordinates of daughter nucleus entry into the plastic. On the other hand, the sensitive layer is thick enough to ensure a substantial UCN detection efficiency ($\sim 30\%$). The measuring technique and the preliminary analysis of results were given in Ref. [47] and will be discussed in detail in subsequent publications. Here we only demonstrate the feasibility of this technique and discuss the preliminary result (Fig. 2) of the first direct measurement of the neutron density above the mirror with a spatial resolution of $1-2$ μm [48]. In Fig. 2: (i) the neutron wave functions in quantum states are known [formula (3)]; (ii) the quantum level populations measured with the aid of the position-sensitive detector coincide with the result of an independent measurement by the technique of two scatterers [36]; (iii) the spatial detector resolution is assumed to be perfectly high; the zero height above the mirror was determined via a fit procedure from the experimental data. A comparison of the experimental data with the theoretical prediction in Fig. 2 suggests: (i) the measured residence density distribution for neutrons above the mirror on the whole corresponds closely to the theoretical prediction; (ii) the spatial detector resolution can be estimated, for instance, using the steepest portion of the dependence near the zero height. It is equal to 1.5 μm; (iii) even a relatively small neutron density variation of $\sim 10\%$, which is expected for a mixture of several quantum states employed in the present experiment, can be measured by this technique.

However, the measurement presented in Fig. 2 is merely a test of the detector for spatial resolution and is not optimized for studying the neutron quantum states in this system. In Ref. [22], the measurement with the position-sensitive detector was analyzed from the standpoint of its optimization for the identification of neutron quantum states. Figure 3 depicts the probability $\psi_n^2(z)$ of neutron detection at a height z above the mirror surface for four pure quantum states. Clearly, each dependence $\psi_n^2(z)$ has n maxima and $n-1$ minima between them with zero values at the minima, which is characteristic of any standing wave. An ideal experiment would consist of the extraction of one or several pure quantum states higher than the first one ($n > 1$) and the direct measurement of neutron detection probability against the height above the mirror with the aid of a position-sensitive detector with a spatial resolution of ~ 1 μm. Let us consider a possible method for realizing such an experiment. One or two lower quantum states can be selected with a scatterer by the conventional method adopted in all of our previous experiments, which showed that the spectrometer resolution is sufficient for this. The method of transferring neutrons from

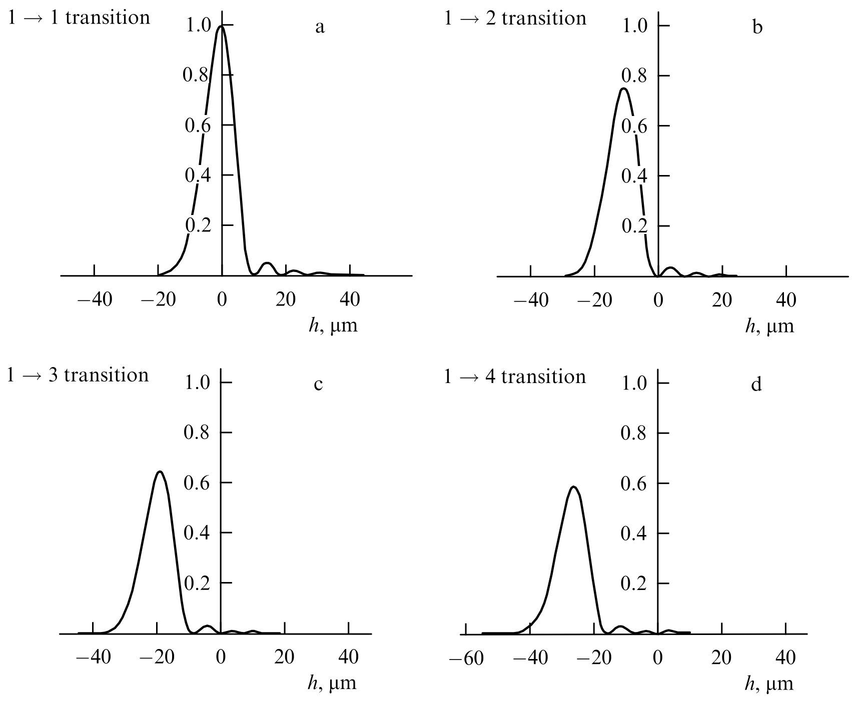

Figure 5. Probability of neutron transition from the 1st quantum state, prior to transit through the step, to the 1st (a), 2nd (b), 3rd (c), and 4th (d) quantum states on transit through the step as a function of the step height h.

the lower quantum states to the higher states was considered in Ref. [22]. It involves the fabrication of a small negative step on the lower mirror, as shown in Fig. 4. Neutrons are in quantum states both to the left of the step and to the right of it. However, the corresponding wave functions turn out to be shifted relative to each other by the step height $\Delta z_{\rm step}$. In their passage through the step, neutrons are redistributed from the nth quantum state prior to the step $\psi_n^{\rm before}(z + \Delta z_{\rm step})$ over the quantum states $\psi_k^{\rm after}(z)$ after the step with some weights $\beta_{nk}^2(\Delta z_{\rm step})$. In this case, the step can be treated as an infinitely fast perturbation and therefore the transition matrix element $\beta_{nk}(\Delta z_{\rm step})$ is

$$\beta_{nk}(\Delta z_{\rm step}) = \int_{-\infty}^{\infty} \psi_n(z + \Delta z_{\rm step})\, \psi(z)\, {\rm d}z\,. \qquad (4)$$

Figure 5 shows the probability β_{1k}^2 of transition from the 1st quantum state, prior to passing through the step, to the 1st, 2nd, 3rd, and 4th quantum states after passing through the step.

When the negative step is large enough, for instance is equal to (-15 μm), the likelihood β_{11}^2 of neutron detection in the lower quantum state after passing through the step is negligible. The similar probability β_{n1}^2 for neutron transitions from higher initial states is also low. Any overlap integral $\beta_{n1}(-15\ \mu{\rm m})$ is small, for the spatial dimension of the neutron wave function in the lower quantum state $\psi_1(z)$ is smaller than 15 μm.

Figure 6 shows the probability of neutron detection above the mirror depending on the height after the neutron passes through the negative 15-μm step. The probability is plotted in two cases: for one and two quantum states in front of the step. It is evident that the expected spatial variation of neutron density is clearly defined and can be measured. The reason of so strong a neutron density variation in the case of elimination of the lower quantum state is simple: one can see from Fig. 3 that only the lower quantum state has a peak near 10 μm. The remaining low-lying quantum states possess a minimum at this height. Therefore, several lower quantum states ($n > 1$) are 'coherently' combined: the probability of neutron detection at a height of ~ 10 μm is systematically much lower than for neighboring heights. This characteristic behavior of the neutron wave functions in the quantum states in the gravitational field above the mirror, as well as the successful first testing of the position-sensitive detector with a uranium coating, gives promise that neutron quantum states will be possible to identify by way of directly measuring the above-mirror neutron detection probability with the use of the position-sensitive detector.

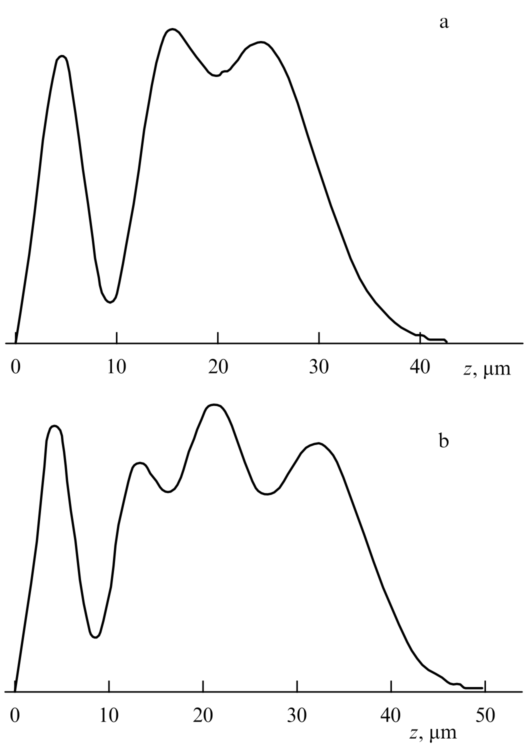

Figure 6. Probability of neutron residence versus height above the mirror on neutron transit through a negative 15-μm step for two cases: one (a) and two (b) lower quantum states prior to the passage through the step.

5. Conclusion

This review outlines the results of investigating the neutron quantum states in the potential well formed by the terrestrial gravitational field and a horizontal mirror. The new, more precise measurements confirm the very existence of state quantization for neutrons and allow estimating the characteristic dimensions of their eigenfunctions.

Such an experiment may be sensitive to hypothetical additional fundamental short-range forces. In particular, even the very existence of neutron quantum states in such a system makes it possible to somewhat improve the existing limit corresponding to a distance of the order of nanometers in the theories with three additional spatial dimensions.

The technique of direct measurement of above-mirror neutron detection probability distribution with the aid of a position-sensitive detector has been shown to offer several advantages. As a differential measurement technique, statistically it is the most exact. At the same time, methodically it is supposedly the most reliable, for it does not make use of a scatterer, which inevitably deforms the quantum states and lowers the accuracy of measurement. The demonstrated characteristic behavior of the neutron wave functions in the quantum states in the terrestrial gravitational field above the mirror at a distance of ~ 10 μm from the mirror surface, as well as the successful first testing of the high-resolution position-sensitive detector with a uranium coating, gives promise that this technique will enable identification of the neutron quantum states in the gravitational field above the mirror.

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