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Search for quantum states of the neutron in a gravitational field: gravitational levels

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Abstract

The neutron could occupy quantum stationary states if it is trapped between the Earth's gravitational field on one side and the Fermi quasi-potential of a mirror on the other side. The quantum states cause a strong variation in neutron density, both for separate energy levels and for a mixture of low-energy states. The use of a position sensitive UCN (ultracold neutron) detector allows simultaneous measurement of the position probability density distribution in the total range of interest and increases significantly the statistics, making possible such an experiment. In this article we describe a specially developed neutron spectrometer and a method of measurement of such quantum states. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The measurement of quantum effects in a gravitational field puts serious constraints on any type of experiment. The quantum effects are negligible for macroscopic objects. Charged particles undergo strong electromagnetic interaction. A neutral particle with a long lifetime is convenient. Luschikov and Frank discussed a measuring scheme for such an experiment with a neutron and mentioned the main methodical problems [1,2]. A relatively easy method for the measurement of such states is discussed in Ref. [3]. The energy range of UCN (ultra-cold neutrons) allows installations of reasonable size, and available neutron fluxes ensure reasonable statistics. Probably the measurement of quantum states is possible also with an atom. Any quantum effect in a gravitational field has not been measured yet. We have developed a high-precision one-component neutron gravitational spectrometer for such studies.

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2. Mathematical description

The potential energy of a neutron trapped between the Earth's gravitational field and a positive surface potential barrier is

$$U(z) = \begin{cases} \infty & \text{for } z < 0, \\ mgz & \text{for } z \geq 0. \end{cases} \tag{1}$$

In the energy range of interest the Fermi quasi-potential can be considered as infinitely high. The potential $U(z)$ is uniform in the horizontal plane (x, y) if the bottom mirror is flat and horizontal. Such a potential causes quantum stationary states which are described by a one-dimensional (z) Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}z^2} + mgz\psi = E\psi. \tag{2}$$

The general solution for such an equation is

$$\psi = A\phi(\xi). \tag{3}$$

$\phi(\xi)$ is an Airy function, A is a normalization coefficient.

Energy levels correspond to the solution of the following equation:

$$\phi\left(-\frac{\sqrt[3]{2}}{\sqrt[3]{mg^2\hbar^2}}E\right) = 0. \tag{4}$$

Stationary state wave functions for the corresponding energy levels $\psi_n(z)$ are the following (n is the number of a gravitational level):

$$\psi_n(z) = A_n(z)\phi\left(\frac{z}{z_0} - \alpha_n\right) \tag{5}$$

where $\phi(z/z_0 - \alpha_n)$ is an Airy function, $z_0 = 3\sqrt{\hbar^2/2m^2g}$ is a length scale for the quantum levels, α_n are the corresponding roots,

$$A_n = \frac{1}{\sqrt{z_0\int_{-\alpha_n}^{\infty}\phi(\zeta)^2\,\mathrm{d}\zeta}}.$$

Energies of four lowest levels are 1.44, 2.53, 3.42 and 4.21 peV. These quantum levels are shown in Fig. 1. The neutron density as a function of height is proportional to the square of the neutron wave function $\psi_n^2(z)$. These functions have n maxima and $n-1$ minima with zero probability in each minimum as for any ordinary standing wave.

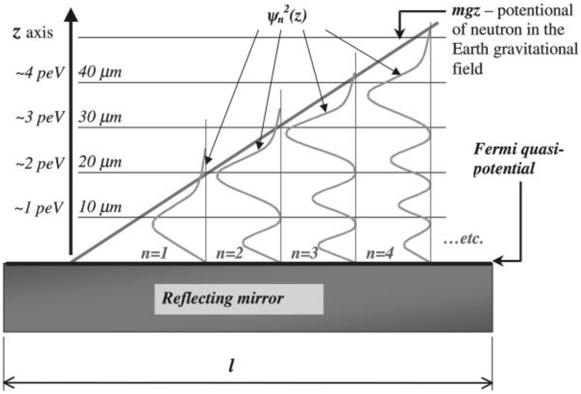

Fig. 1. Quantum states of a neutron in a trap between the Earth's gravitational potential and a Fermi quasi-potential.

The "classical" estimate (the population is proportional to $E^{1.5}$) for the height-density $\psi^2(z)$ for a mixing of different quantum states is

$$\psi^2(z) = \frac{1}{N}\frac{\sum_{n=1}^{N}\psi_n^2(z)(E_{n+1}^{1.5} - E_{n-1}^{1.5})}{\sum_{n=1}^{N}(E_{n+1}^{1.5} - E_{n-1}^{1.5})}. \tag{6}$$

As seen from Fig. 2, the neutron density variation is considerable for a mixture of several quantum states. Moreover, it can be strongly enhanced near the surface by cutting out one lowest level.

3. Measuring scheme

The general scheme of the experiment is shown in Fig. 3. The horizontal velocity component ~ 5 m/s in the beam is much larger than the vertical one (< 5 cm/s). Therefore, in a vertical slice one can find several neutron "planar jets" with a density depending on the distance from the bottom mirror. These jets are homogenous in the horizontal plane.

To select few lower levels we place an absorber/scatterer above the bottom mirror, for example, at a height of 50 μ, which corresponds to the 5th level. Neutrons are non-specularly scattered from its rough surface and then lost in it due to the negative (or better still small negative) critical energy and large loss cross-section. A good candidate

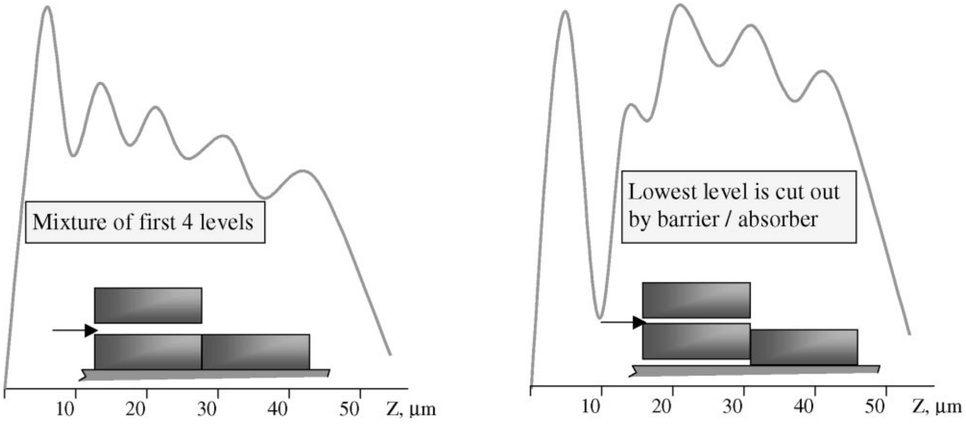

Fig. 2. Mixture of quantum states.

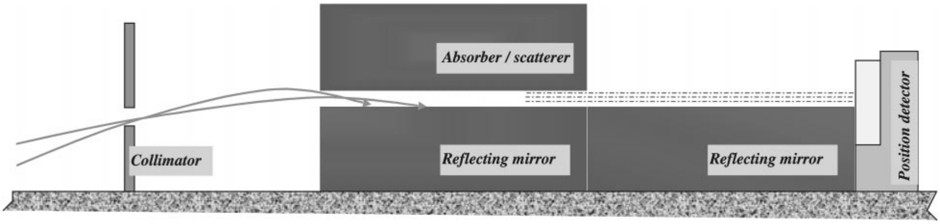

Fig. 3. General scheme of the experiment.

for such an absorber/scatterer is a thin, rough polyethylene foil fixed to a flat glass surface.

A constant potential $U(z)$ (Eq. (1)) has to act on a neutron for at least 6.6 ms to provide the energy resolution of 10^{-13} eV as follows from the uncertainty principal $\Delta T \approx \hbar/\Delta E$. This would correspond to a jumping height in the Earth's gravitational field of 1 μm. Neutrons with 5 m/s velocity travel 3.3 cm in that time period. On the other hand, the "classical" time interval between two consecutive collisions with the bottom for a neutron in the first level would be $\Delta T_2 = 2\sqrt{2h_1/g} \approx 3.4 \times 10^{-3}$ s. So, after very few collisions the energy levels are well resolved. Therefore we use UCN in a "flow through" mode bearing in mind that the installation size should not be too large. In the absence of angular mixing (specular reflections from an ideally flat and horizontal surface) horizontal and vertical motions are independent. That gives a continuous spectrum in the horizontal plane but quantum states for vertical motion.

In fact, Eq. (2) is different for our conditions because the absorber/scatterer modifies strongly the higher-energy quantum states. However, it cuts them out in an ideal case while this change is small for the low-energy levels. Therefore the presented solution should describe well the final density distribution.

4. UCN detector

For the present experiment we developed a special UCN detector with high space resolution and absolute positioning of about 1 μm (Fig. 4). This is a standard plastic nuclear track detector coated with a thin ^{235}U layer. Absorption of UCN in the uranium causes nuclear fission in 85% of the cases. Usually two daughter nuclei are emitted in

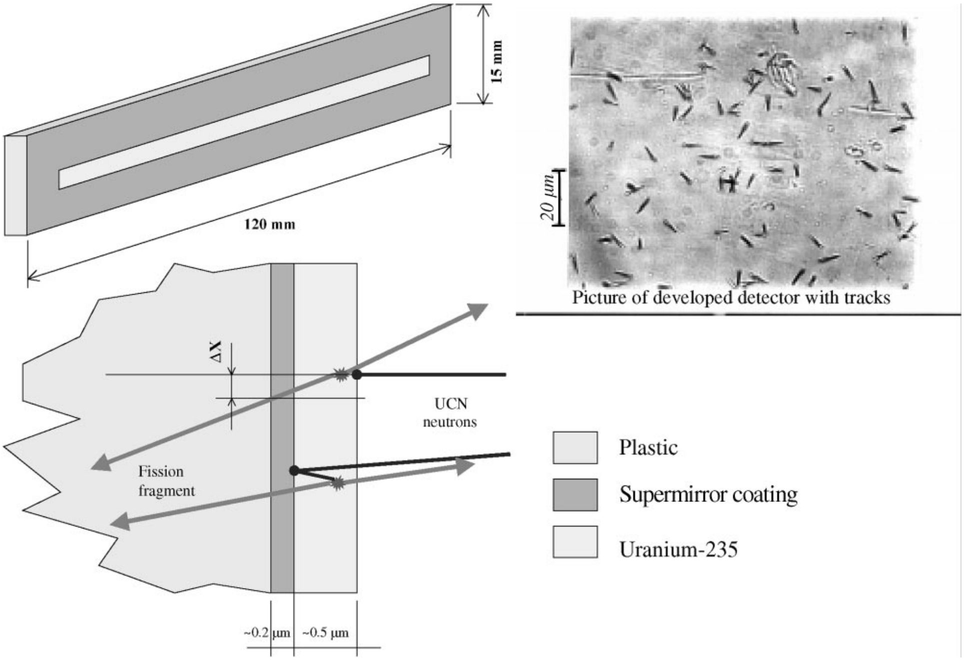

Fig. 4. Position sensitive detector for ultra-cold neutrons.

opposite directions, one of them towards the track detector. Chemical treatment allows us to increase the diameter of the corresponding track up to 1 μm, and then it is visible with a scanning microscope.

The converting uranium coating is made thin enough such that the point of entrance of a daughter nucleus into the track detector is not too far from the absorption point. On the other hand the layer thickness should be large enough to provide high detection efficiency. Fortunately, both requirements are satisfied reasonably well for a pure ^{235}U coating thickness of about 0.5 μm. In order to further increase the efficiency, we use a thin sublayer between the uranium and the track detector. It has high critical velocity (supermirror) allowing UCN to cross the uranium layer twice due to back reflections from the sublayer.

The background for such a detector is low. Good separation from background events is provided by the high energy released in the fission of the ^{235}U nucleus. Also the small vertical height (about 50 μm) of the sensitive window of the detector ensures a small total surface area from which one counts the background.

5. Experimental installation

The drawing of the experimental installation is shown in Fig. 5.

Vibrations could cause parasitic transitions between levels and "smearing" of the quantum states due to the Doppler effect. Therefore, the complete assembly is mounted on a standard passive pressurized anti-vibration table. A system of three interconnected pneumatic valves provides approximate level constancy for the "floating" table top. A polished flat granite slab is mounted to the "floating" table top on three active legs consisting of piezo translators. They are connected into a closed loop with precise inclinometers installed on the stone surface. Length variations of the piezo translators allow automatic active leveling for the stone with an absolute precision better than 10 μrad.

A vacuum chamber with a thin aluminum entrance window avoids neutron losses in air over the relatively long path inside the device. A special antimagnetic screen insures that the influence of magnetic field gradients on the neutrons is much smaller than that of the Earth's gravitational field.

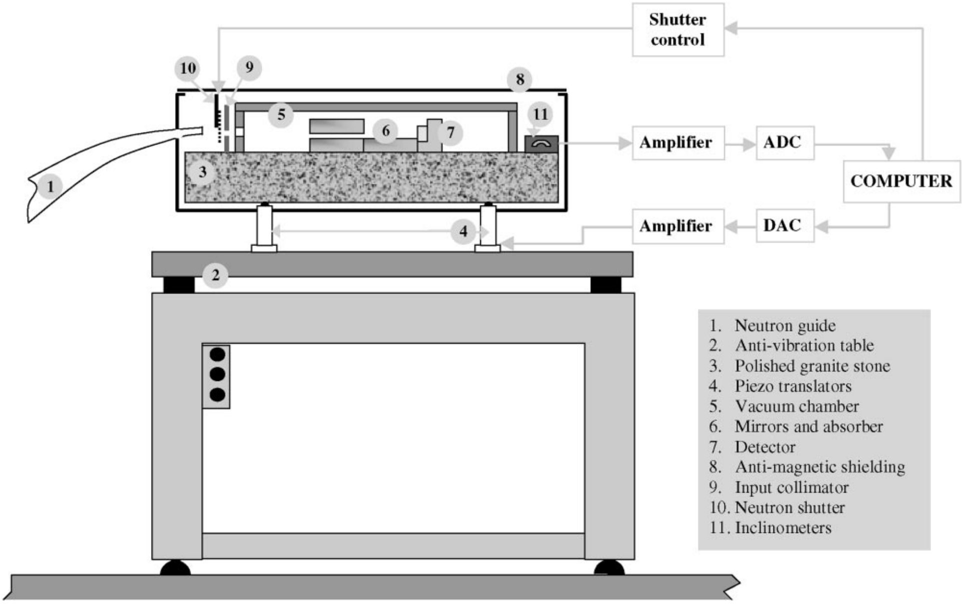

Fig. 5. The experimental setup.

The bottom mirrors are produced and adjusted with an accuracy of 1 μ required by the range of the inter-level distances. The UCN detector is mounted near to the edge of a bottom mirror to have no neutron beam travelling free of potential. This prevents spreading of the neutron "plane jets" due to angular dispersion in the "plain jets". The vertical z-component for the neutron spectrum is shaped by the polyethylene absorber/scatterer height, the x-component (perpendicular to the detector surface) by a horizontal collimating slit, the y-component by the sizes and relative positions of the entrance slit, the mirrors and the detector.

6. Estimations of statistics with a "pilot" version of the installation

A UCN density of up to $0.05 \mathrm{n/cm^3/(m/s)^3}$ (in a velocity range < 6.2 m/s) can be provided at the entrance slit. The corresponding maximum possible neutron flux is about 3 n/s if the slit height (between the bottom mirror and the absorber/scatterer) is 0.005 cm, the detector and the mirror widths are 10 cm, the mirror length is 3 cm and the velocity range is 5.2–7 m/s. For a factor of 10 loss in flux within the device and for a detector efficiency of 10% the expected counting rate of 0.03 n/s is still much higher than the background. To measure the height density distribution with a position resolution of 1 μ over 50 points and statistical accuracy of 5% in each point one needs 5–10 days. Better detector efficiency, wider neutron spectrum and/or bigger mirrors and detector would allow us to improve the statistics. But even the present estimation seems to be promising.

7. Conclusion

This installation is a general-purpose one-component gravitational spectrometer with the best currently available energy and space resolution. If the quantum stationary states of a neutron in a gravitational field would be found experimentally, numerous experiments with resonance transitions of a neutron from one level to another and measurements using magnetic fields could be considered. A modified installation could be used to lower the upper limit for the neutron charge. The feasibility of such an experiment depends on the results of mirror and quasi-mirror reflection studies.

We have not discussed here in detail any possible methodical problems in the described method of measurement of neutron quantum states in the Earth's gravitational field but we hope to do the first experimental tests soon.

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